

Harmonic Synthetic Aperture Radar Processing

Tor Berger and Svein-Erik Hamran

Abstract—We review the basic concept of Frequency Modulated Continuous Wave (FMCW) and describe how such a radar system can be used in a harmonic radar concept. It is argued that synthetic aperture radar (SAR) for FMCW harmonic radar can be implemented by carefully choosing the wavenumbers in the mixing part of the FMCW concept. It is also argued that the following SAR processing is an extension of conventional SAR processing when applied to a harmonic FMCW system.

Index Terms — Frequency Modulated Continuous Wave, Harmonic Radar, Synthetic Aperture Radar.

I. INTRODUCTION

THE basic idea behind harmonic radar is to exploit the nonlinearities of the electromagnetic properties of a reflecting target causing higher order harmonics in the radar return.

A concept for harmonic radar was developed for avalanche rescue [1], and a commercial detector was introduced by the Swedish company Recco in 1983 [2]. This was a single tone system receiving at twice the transmitting frequency. Harmonic radar has also been used to track insects and small animals [3]-[7]. Other known applications are for example in problems related to detection of vital signs [8]-[10], temperature sensing [11]-[12] and detection of RF electronics [13]. Many of the applications for harmonic radar rely on using tags that cause higher order harmonic reflections when illuminated by an RF signal. Often these tags contain a nonlinearity such as a Schottky diode. Other targets, e.g. RF electronics, could cause the same effect due to nonlinearities in their design.

Harmonic radar prototypes and systems have been developed for security and military applications since almost 50 years, mainly for detecting concealed electronics and weapons [14]. The powerful METRRA radar was developed by IIT research institute for the U.S. Army. Theoretical work has been carried out on detection of in-foliage nonlinear scatterers [16], and in recent years work has also been done on cognitive nonlinear radar by the U.S. Army [17]. Smaller systems are now available commercially, such as the REI Orion [18] and Winkelmann Hawk [19].

In radar, range resolution is inversely proportional to effective range bandwidth [20]. Cross range information requires transmit of multiple pulses at different aperture positions. In a synthetic aperture radar system one can obtain 2D and 3D resolution of a scene [21], and thereby in a harmonic radar concept it would be possible to locate the position of objects causing higher order harmonic returns.

In an FMCW system we may transmit at one frequency band; f_{low} - f_{high} and receive at another frequency band, for example $2f_{low}$ - $2f_{high}$. By doing this we isolate any second order harmonics of the radar return. Unwanted clutter does normally not contain higher order harmonics, meaning that the signal to clutter ratio can be increased for signals containing higher order harmonics.

There are some hardware requirements to be met when designing harmonic radar. The antennas used must cover both the fundamental frequency band and the harmonics, meaning for all purposes that they must be wideband. The same applies for the amplifiers. Also, since we seek harmonics generated from the return signal from a target, we must ensure that the transmitted signal contains little or no higher order harmonics. One way of doing this is to split the transmitted signal and apply a low pass filter on the signal fed to the antenna [22].

Although the signal to clutter ratio is increased, there are signal to noise ratio (SNR) limitations for the low signal levels anticipated from higher order harmonics [14], meaning that harmonic radars mostly would be short range systems.

In this paper we review the basic concept of FMCW and the extra requirements added when introducing the harmonic radar concept. We argue that synthetic aperture radar (SAR) processing of the return of harmonic FMCW radar is an extension of conventional SAR processing.

II. FREQUENCY MODULATED CONTINUOUS WAVE

In FMCW radar a linear frequency modulated signal is transmitted. The reflected signal from targets in the scene is received and mixed with a replica of the transmitted signal, and then low pass filtered. The resulting signal contains beat frequencies indicating the range to targets.

Although there is no sole inventor of FMCW, many people have contributed significantly to the development of the technique, for example [23]-[26]. Other terms used for the mixing part of FMCW are for example “stretch” [27] or “de-ramp”.

Fig. 1 shows the basic concept of FMCW. A signal containing a frequency sweep over a bandwidth B is transmitted. The sweep time is T_s . A target at range r causes a reflected signal arriving at the round trip time $\tau = 2r/c$, where c is the propagation velocity of the medium.

Manuscript submitted for review on April 23, 2015.

T. Berger is with the Norwegian Defence Research Establishment (FFI), 2027 Kjeller, Norway (e-mail: Tor.Berger@ffi.no).

S.-E. Hamran is with the Norwegian Defence Research Establishment (FFI), 2027 Kjeller, Norway, and also with the Department of Informatics, University of Oslo, Blindern, 0316 Oslo, Norway.

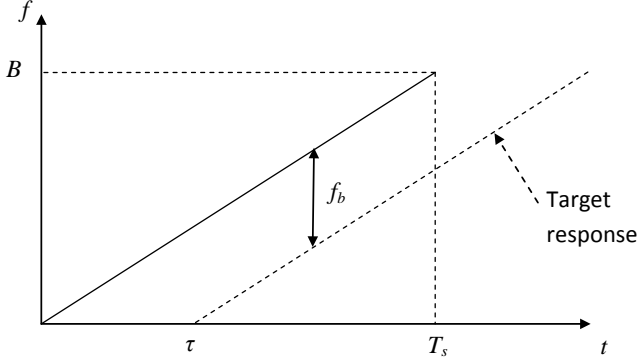


Fig. 1 FMCW concept. A linear frequency modulation sweep is transmitted at $t = 0$. The reflected signal from a target is received at $t = \tau$. The difference in frequency, f_b , is used to estimate the range to the target.

Assuming the frequency range of the transmitted signal is $[f_0, f_0 + B]$, and $\omega_0 = 2\pi f_0$, the transmitted signal is then

$$s_0 = \rho_0 \cos \left[\omega_0 t + \frac{A}{2} t^2 \right] \quad (1)$$

where $A = 2\pi B/T_s$. The return signal is shifted in time, and altered in amplitude, i.e.

$$s_1 = \rho_1 \cos \left[\omega_0 (t - \tau) + \frac{A}{2} (t - \tau)^2 \right]. \quad (2)$$

The transmitted and returned signal are mixed, i.e. multiplied, in the receiver, and (3) shows the expanded result of the mixing.

$$s_m = \frac{\rho_0 \rho_1}{2} \left[\cos \left\{ (2\omega_0 - A\tau)t + At^2 + \left(\frac{A}{2} \tau^2 - \omega_0 \tau \right) \right\} + \cos \left\{ A\tau t + \left(\omega_0 \tau - \frac{A}{2} \tau^2 \right) \right\} \right] \quad (3)$$

The first term of (3) is filtered out and the remaining term describes a beat signal at constant frequency. The beat frequency is the derivative of the phase, i.e.

$$f_b = \frac{A\tau}{2\pi} = \frac{B\tau}{T_s}. \quad (4)$$

Remembering $\tau = 2r/c$, where r is the range to the target, we see that the range and beat frequency is related by the expression

$$r = \frac{cT_s}{2B} f_b. \quad (5)$$

In a practical system, the sampling of the mixed signal is limited by the constraints of the hardware. The sampling frequency, f_s , is typically in the order of MHz, while the signal carrier could be in the order of GHz. The maximum unambiguous range is given by (5) by letting the maximum beat frequency be equal to half the sampling frequency f_s .

We also know that the frequency resolution of a signal is inversely proportional to the signal duration. In this context it gives us the beat frequency resolution as $\Delta f_b = 1/T_s$. The range

resolution is directly proportional to the beat frequency resolution, and from (5) we have

$$\Delta r = \frac{c}{2B}. \quad (6)$$

III. HARMONIC FMCW

As in a conventional FMCW system, a sweep signal covering a frequency band $[f_0, f_0 + B]$ is transmitted (see (1)). We now assume that nonlinearities in the reflective properties of the target causes second order harmonics. As in the previous section we assume a single target at range r . The reflected signal is time delayed, altered in amplitude, and it also contains a second order harmonic, as shown in (7).

$$s_H = \rho_1 \cos \left[\omega_0 (t - \tau) + \frac{A}{2} (t - \tau)^2 \right] + \rho_2 \cos [2\omega_0 (t - \tau) + A(t - \tau)^2] \quad (7)$$

The received signal in (7) is mixed with the transmitted signal in (1), and the expanded result is

$$s_m^1 = \frac{\rho_0}{2} \left[\begin{array}{l} \rho_1 \cos \left\{ (2\omega_0 - A\tau)t + At^2 + \left(\frac{A}{2} \tau^2 - \omega_0 \tau \right) \right\} \\ + \rho_1 \cos \left\{ A\tau t + \left(\omega_0 \tau - \frac{A}{2} \tau^2 \right) \right\} \\ + \rho_2 \cos \left\{ (3\omega_0 - 2A\tau)t + \frac{3}{2} At^2 + (A\tau^2 - 2\omega_0 \tau) \right\} \\ + \rho_2 \cos \left\{ (\omega_0 - 2A\tau)t + \frac{A}{2} t^2 + (A\tau^2 - 2\omega_0 \tau) \right\} \end{array} \right] \quad (8)$$

The first two terms in (8) are equal to (3) The last two terms of (8) are filtered out in the receiver, provided there is no overlap in frequency with the beat signal. This means that the minimum frequency of the last term must be greater than the maximum beat frequency given by the second term of (8). This put restriction on the maximum possible time delay given by

$$f_0 - \frac{2B}{T_s} \tau_{max} = \frac{B}{T_s} \tau_{max} \Rightarrow \tau_{max} = \frac{f_0 T_s}{3B}. \quad (9)$$

This means that a second order harmonic does not affect the output of a conventional FMCW system. The same can be shown when harmonics higher than order two is introduced in the return signal.

Now consider the case where the return signal is mixed with a signal at twice the transmitted frequency content. In this case the return signal given by (7) is mixed with the signal

$$s_{0,H} = \rho_0 \cos [2\omega_0 t + At^2]. \quad (10)$$

The expanded result of the mixing with the signal in (10) is

$$S_m^2 = \frac{\rho_0}{2} \begin{bmatrix} \rho_1 \cos \left\{ (3\omega_0 - A\tau)t + \frac{3}{2}At^2 + \left(\frac{A}{2}\tau^2 - \omega_0\tau \right) \right\} \\ + \rho_1 \cos \left\{ (\omega_0 - A\tau)t + \frac{A}{2}t^2 + \left(\omega_0\tau - \frac{A}{2}\tau^2 \right) \right\} \\ + \rho_2 \cos \left\{ (4\omega_0 - 2A\tau)t + 2At^2 + (A\tau^2 - 2\omega_0\tau) \right\} \\ + \rho_2 \cos \left\{ 2A\tau t + (2\omega_0\tau - A\tau^2) \right\} \end{bmatrix} \quad (11)$$

The first three terms are filtered out in the receiver, and the last term represents a beat frequency of twice the value compared to the conventional case. But the bandwidth has been doubled in this case, meaning that the range given by (5) is not changed. It should be noted, however, that when applying the harmonic concept with FMCW, care must be taken to maintain the unambiguous range. If a fixed unambiguous range is required, and the second harmonic is investigated, the sampling frequency of the beat signal must be doubled as compared to the conventional case. Accordingly, if harmonics up to order L is investigated, the sampling frequency must be increased by a factor L.

We note that after low pass filtering the result indicated in (11), the overall result is the same as if a conventional FMCW system had been applied to a single harmonic signal within the frequency band $[2f_0, 2(f_0+B)]$. This means that the different harmonic components of the return signal is separated in the receiver. This will become useful when applying a synthetic aperture processor to the signal.

One benefit of using harmonic radar is its clutter suppression capabilities. However, the signal to noise ratio (SNR) of the harmonic returns is lower than for the fundamental return. The received power from the fundamental return follows an inverse fourth power law with respect to range, while the second and third harmonics re-radiated signals follow inverse sixth and eight power laws, respectively [14]. This means that the transmitted power should be increased, or the harmonic radar system should be used at short standoff. Increasing the number of aperture positions of a synthetic aperture would improve the situation. As an example, 1000 aperture positions would give 30 dB processing gain in a SAR processor.

IV. SYNTHETIC APERTURE RADAR PROCESSING

In synthetic aperture radar return signals collected along a trajectory are processed to increase the cross range resolution. According to Fig. 2, we assume a radar travelling along the positive x-axis, looking in the y-direction. A 2D geometry is assumed for simplicity. For 3D geometry we replace y with $(h^2+y^2)^{1/2}$, where h is the height above ground. Again, for simplicity, a single reflection is assumed. A scene consisting of several reflectors is formed by adding the contribution from each reflector. For the Harmonic SAR described here there will be no practical difference between using time domain backprojection [21] or wavenumber algorithm (ω - k) [28]-[29] as SAR processor. For the sake of illustration, ω - k is used in the following.

The transmitted signal may be seen as the integration of single frequencies swept across the bandwidth. The back scattered field caused by a point scatterer at some distance r is

the product of the transmitted field from the antenna to the point scatterer and the reflected field from the point scatterer to the antenna, as shown in (12).

$$S(k) = \rho e^{-jkr} e^{-jkr} = \rho e^{-j2kr} \quad (12)$$

In eq. 12 ρ is the overall strength of the scattered field and k is the wavenumber; $k=2\pi f/c$. The range to the target is given by

$$r = \sqrt{(x-x_p)^2 + y_p^2}. \quad (13)$$

Taking the Fourier transform of (12) along the x-axis we obtain the scattered field in the k_x domain;

$$S(k_x, k) = \rho \int_{-\infty}^{\infty} e^{-j2k\sqrt{(x-x_p)^2 + y_p^2}} e^{jk_x x} dx \quad (14)$$

Solving (14) with the stationary phase principle, we end up with (except for a constant phase term)

$$\tilde{S}(k_x, k) = e^{-j \left[k_x x_p + \sqrt{4k^2 - k_x^2} y_p \right]} \quad (15)$$

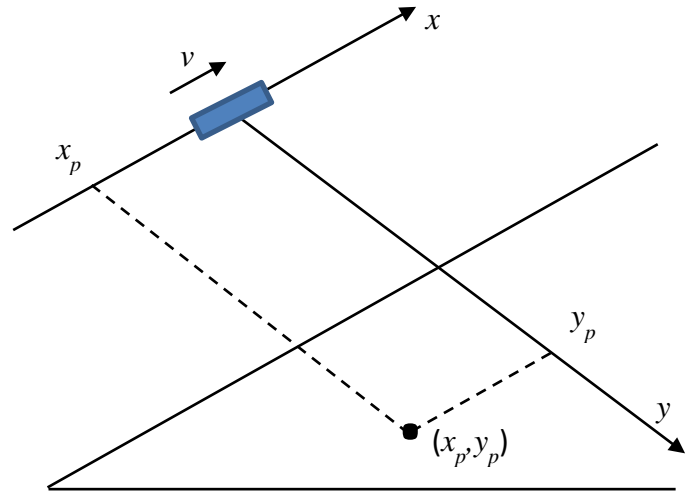


Fig. 2 Synthetic aperture radar geometry

Defining $k_y = \sqrt{4k^2 - k_x^2}$ it is seen that the scene can be reconstructed by taking the 2D inverse Fourier transform of (14), as shown in (16).

$$s(x, y) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} \tilde{S}(k_x, k_y) e^{-j[k_x x + k_y y]} dk_x dk_y \quad (16)$$

In order to use FFTs in the reconstruction, the observed signal must be resampled to a rectangular grid, which for example is done in the ω - k algorithm. Otherwise, one could evaluate (16) directly on the existing grid points defined by k_x and k_y .

V. HARMONIC SAR

As described earlier, in the harmonic radar concept we assume that the backscattered field from scatterers contains higher order harmonics. The higher order harmonics add to the

signal consisting of the first harmonic. If we again consider the single frequency case, and a total of L harmonics, the backscattered field from a point scatterer at range r can be written as

$$S(k) = \rho_0 e^{-jk r} [\rho_1 e^{-jkr} + \rho_2 e^{-j2kr} + \rho_3 e^{-j3kr} + \dots + \rho_L e^{-jLkr}] \quad (17)$$

The term in front of the parentheses is the incoming field from the antenna to the point scatterer, while the terms inside the parentheses are the harmonics of the reflected field. We see from (17) that the observed field is the sum of individual reflected fields with different wavenumbers; $k, 1.5k, 2k, \dots$ and so on.

Applying an FMCW system for SAR processing we know from previous sections that mixing the reflected signal with a higher order harmonic signal will isolate the harmonic in question. This means that for SAR processing of harmonic signals, we simply have to change the wavenumber k in the reconstruction algorithm. Otherwise the processing is identical

to the conventional case. The reconstruction of harmonic images is illustrated in Fig. 3 where the reflected signal is mixed with signals of increasingly higher frequency bands. To improve the signal to clutter ratio harmonic images can be summed coherently. If we are interested in inspecting all harmonics higher than 1, we sum harmonic images 2 – L .

Targets may be moving, and one could expect different smearing in the different harmonic SAR images. Inspecting the general azimuth displacement one finds that it depends on the velocity vectors of the target and the SAR platform [30], [31]. Raney [30] showed that a radial velocity of the target produces a range smear independent of the signal wavelength. Also, the effect of increased Doppler of a moving target at higher order harmonics is cancelled by the increased center frequency. In the SAR scheme shown in Fig. 3, the receiver at each harmonic band is matched to the return signal, meaning that effects of target motion would be the same as for the fundamental band.

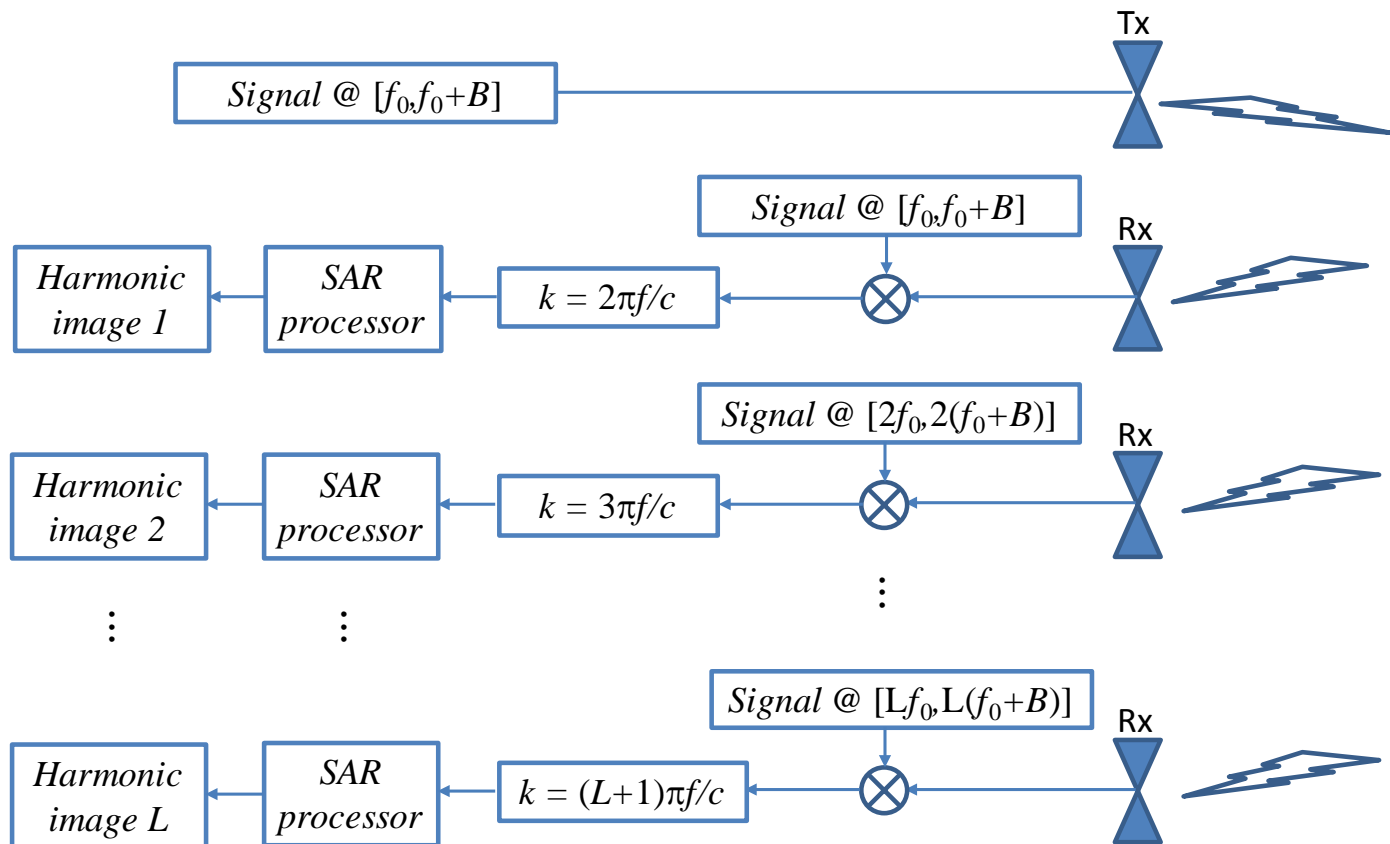


Fig. 3 Schematics of harmonic SAR processing

VI. CONCLUSIONS

By applying the harmonic radar concept to FMCW synthetic aperture radar it will be possible to localize the sources of higher order harmonic signal returns in a scene. This could be of interest in crowded scenes, both for tracking

small objects with tags on, or detecting non-cooperative targets. It has been justified that it will be possible to construct harmonic FMCW SAR, and perform SAR processing on signal returns. The next step would be to implement such a system in hardware, and do the SAR processing.

REFERENCES

- [1] P. Fuks, "Harmonic Radar, a Modern Method for Location of Avalanche Victims," *Ph. D dissertation*, Division of Electromagnetic Theory, Royal Inst. Of Technology, Stockholm, Sweden (1981).
- [2] www.recco.com, Lidingsö, Sweden.
- [3] G. L. Lövei, I. A. N. Stringer, C. D. Devine, and M. Cartellieri, "Harmonic Radar – A Method Using Inexpensive Tags to Study Invertebrate Movement on Land," *New Zealand Journal of Ecology*, vol. 21, no. 2, pp. 187-193, 1997.
- [4] B. G. Colpitts, and G. Boiteau, "Harmonic Radar Transceiver Design: Miniature Tags for Insect Tracking," *IEEE Trans. Antennas and Propagation*, vol. 52, no. 11, pp. 2825-2832, November 2004.
- [5] Z-M. Tsai, P-H. Jau, N-C. Kuo, J-C. Kao, K-Y. Lin, F-R. Chang, E-C. Yang, and H. Wang, "A High-Range-Accuracy and High-Sensitivity Harmonic Radar Using Pulse Pseudorandom Code for Bee Searching," *IEEE Trans. Microwave Theory and Techniques*, vol. 61, no. 1, pp. 666-675, January 2013.
- [6] H. Aumann, E. Kus, B. Cline, and N. W. Emanetoglu, "A Low-cost Harmonic Radar for Tracking Very Small Tagged Amphibians," *IEEE Int. Instrumentation and Measurement Technology Conference (I2MTC)*, Minneapolis, MN, USA, 6-9 May 2013, pp.234-237.
- [7] H. M. Aumann, and N. W. Emanetoglu, "A Wideband Harmonic Radar for Tracking Small Wood Frogs," *IEEE Int. Radar Conference*, Cincinnati, OH, USA, 19-23 May 2014, pp. 108-111.
- [8] X. Gao, A. Singh, O. Boric-Lubecke, and V. M. Lubecke, "Small-scale Displacement Measurement With Passive Harmonic RF Tag Using Doppler Radar," *IEEE Int. Wireless Symposium (IWS)*, Beijing, China, 14-18 April 2013, pp. 1-4.
- [9] L. Chioukh, H. Boutayeb, W. Ke, and D. Deslandes, "Monitoring Vital Signs Using Remote Harmonic Radar Concept," *European Radar Conference (EuRAD)*, Manchester, UK, 12-14 October 2011, pp. 381-384.
- [10] L. Chioukh, H. Boutayeb, D. Deslandes, and W. Ke, "Noise and Sensitivity of Harmonic Radar Architecture for Remote Sensing and Detection of Vital Signs," *IEEE Trans Microwave Theory and Techniques*, vol. 62, no. 9, pp. 1847-1855, September 2014.
- [11] B. Kubina, J. Romeu, C. Mandel, M. Schüßler, and R. Jakoby, "Design of a Quasi-Chipless Harmonic Radar Sensor for Ambient Temperature Sensing," *IEEE SENSORS*, Valencia, Spain, 2-5 November 2014, pp. 1567-1570.
- [12] B. Kubina, J. Romeu, C. Mandel, M. Schüßler, and R. Jakoby, "Quasi-Chipless Wireless Temperature Sensor Based on Harmonic Radar," *Electronics Letters*, vol. 50, no. 2, pp. 86-88, January 2014.
- [13] G. J. Mazzaro, A. F. Martone, and D. M. McNamara, "Detection of RF Electronics by Multitone Harmonic Radar," *IEEE Trans. Aerospace and Electronic Systems*, vol. 50, no. 1, pp. 477-490, January 2014.
- [14] D. J. Daniels, *EM Detection of Concealed Targets*, Hoboken, NJ, Wiley Series in Microwave and Optical Engineering, 2010.
- [15] R. F. Elsner, "Vehicular variable parameter METRRA system", *Report E6224*, U.S. Army Mobility Equipment Research and Development Center, May 1974.
- [16] R. O. Harger, "Harmonic Radar Systems for Near-Ground In-Foliage Nonlinear Scatterers", *IEEE Trans. Aerospace and Electronic Systems*, vol. AES-12, no. 2, pp. 230-245, March 1976.
- [17] A. Martone, D. McNamara, G. Mazzaro, and A. Hedden, "Cognitive Nonlinear Radar", ARL-MR-0837, January 2013.
- [18] www.reiusa.net, Algood, TN, USA.
- [19] www.winkelmann.co.uk, West Sussex, UK.
- [20] G. M. Brooker, "Understanding Millimetre Wave FMCW Radars", *1st International Conference on Sensing Technology*, Palmerston North, New Zealand, 21-23 November 2005, pp. 152-157.
- [21] J. C. Curlander, and R. N. McDonough, *Synthetic Aperture Radar, Systems and Signal Processing*, Hoboken, NJ, Wiley Series in Remote Sensing, 1991.
- [22] K. A. Gallagher, and R.M Narayanan, "Moving Target Indication with Non-Linear Radar", *IEEE Int. Radar Conference*, Arlington, VA, USA, 11-15 May 2015, pp. 1428-1433.
- [23] A. Meta, P. Hooeboom, and L. P. Ligthart, "Signal Processing for FMCW SAR", *IEEE Trans. Geoscience and Remote Sensing*, vol. 45, no. 11, pp. 3519-3532, November 2007.
- [24] R. Wang, O. Loffeld, H. Nies, S. Knedlik, M. Hägelen, and H. Essen, "Focus FMCW SAR Data Using the Wavenumber Domain Algorithm", *IEEE Trans. Geoscience and Remote Sensing*, vol. 48, no. 4, pp. 2109-2118, April 2010.
- [25] J. J. M. de Wit, A. Meta, and P. Hooeboom, "Modified Range-Doppler Processing for FM-CW Synthetic Aperture Radar", *IEEE Geoscience and Remote Sensing Letters*, vol. 3, no. 1, pp. 83-87, January 2006.
- [26] A. Ribalta, "Time-Domain Reconstruction Algorithms for FMCW-SAR", *IEEE Geoscience and Remote Sensing Letters*, vol. 8, no. 3, pp. 396-400, May 2011.
- [27] W. J. Caputi, "Stretch: A Time-Transformation Technique", *IEEE Trans. Aerospace and Electronic Systems*, vol. AES-7, no. 2, pp. 269-278, March 1971.
- [28] J. Gazdag, and P. Sguazzero, "Migration of Seismic Data", *Proceedings of the IEEE*, Vol. 72, No. 10, pp. 1302-1318, October 1984.
- [29] C. Cafforio, C. Prati, and E. Rocca, "SAR Data Focusing Using Seismic Migration Techniques", *IEEE Trans on Aerospace and Electronic Systems*, Vol. 27, No. 2, pp. 194-206, March 1991.
- [30] R. K. Raney, "Synthetic Aperture Imaging Radar and Moving Targets", *IEEE Trans. Aerospace and Electronic Systems*, vol. AES-7, no. 3, pp. 499-505, May 1971.
- [31] S. Axelsson, "Position correction of moving targets in SAR-imagery", *SAR Image Analysis, Modeling and Techniques VI*, edited by Francesco Posa, *Proceedings of SPIE*, vol. 5236, pp. 80-92, January 2004.



Tor Berger received the MSc and PhD degrees in applications of the wavelet transform in image processing from the University of Tromsø, Tromsø, Norway, in 1992 and 1996, respectively. Since 1996, he has been with the Norwegian Defence Research Establishment, Kjeller, Norway, working in different fields such as weapon effects, security sensors, and signal processing. He is currently involved in radar signal processing related to ultra wideband systems and synthetic aperture radar.



Svein-Erik Hamran received an MSc in Technical Physics in 1984 from the Norwegian University of Science and Technology (NTNU, Trondheim, Norway) and a PhD degree in 1990 in Physics from the University of Tromsø. He worked from 1985 to 1996 at the Environmental Surveillance Technology Programme and was in 1989/90 a Visiting Scientist at CNRS Service d'Aeronomie, Paris, France. From 1996 he has been at the Norwegian Defence Research Establishment working as a Chief Scientist managing radar programs. From 2001 to 2011 he was an Adjunct Professor in Near Surface Geophysics at the Department of Geosciences, University of Oslo. From 2011 he has been an Adjunct Professor at the Department of Informatics at the University of Oslo. He is the Principal Investigator of the Radar Imager for Mars' subsurFAce eXperiment – RIMFAX on the Mars 2020 NASA rover mission and a Co-Principal Investigator on the WISDOM GPR experiment on the ESA ExoMars rover. His main interest is UWB radar design, radar imaging and modeling in medical and ground penetrating radar.