SENSIVITY TO CHANGES IN SOUND SPEED ON PASSIVE BEARING AND RANGE ESTIMATION

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SENSIVITY TO CHANGES IN SOUND SPEED ON PASSIVE BEARING AND RANGE ESTIMATION

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IN ENGLISH:
- Towed array
- Passive ranging
- Bearing
- Sound speed

IN NORWEGIAN:
- Tauet antenne
- Passiv avstandsmåling
- Peiling
- Lydhastighet

THESAURUS REFERENCE:

A passive towed array can easily measure the bearing to a target. Range can be found by at least two methods; measurements of time of arrival differences at three spaced hydrophone or hydrophone clusters or triangulation using bearings from three sub-arrays. The accuracy depends on good knowledge of the sound speed. In this report historical data have been analysed to find the change in recorded sound speed over a 5 meter depth interval, equivalent to measuring the sound speed in the tower of a submarine instead of the same depth as the towed array. The effect on bearing and range estimates from global sound speed variations as well as variation along the array have been analysed, and recommendations presented.
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SENSIVITY TO CHANGES IN SOUND SPEED ON PASSIVE BEARING AND RANGE ESTIMATION

1 INTRODUCTION

The motivation for this study is the TISAM project for the ULA class submarines, and the installation of a new towed array passive sonar. It is questioned whether one should install one or more new sound speed sensors at or near the towed array, or whether present practice with a single sound velocity sensor mounted at the submarines tower is sufficient for high quality calculations of range and bearing.

Possible errors in sound speed may be separated into:
- **Global errors:** The actual sound speed in the water volume near the towed array is different from the measured sound speed. This might be because of the vertical distance between the towed array and the sound velocity sensor, or because of a defect or poorly calibrated sensor.
- **Local errors:** Horizontal variations in sound speed (along the towed array).

In this report, an estimate of maximum vertical sound speed gradient based on available measurements is given. This estimate is used in the sensitivity studies for the range and bearing calculation for global errors in sound speed. Sensitivity studies for local errors were also performed. Two algorithms for range and bearing calculation were evaluated:
- Passive Ranging Method (PRS)
- Triangulation using three sub-arrays
2 MAXIMAL VERTICAL VARIATIONS IN SOUND SPEED OVER A VERTICAL DISTANCE OF 5 METERS.

2.1 Data analysis

2.1.1 Available data and quality control

The available data for this analysis were:

- 220 CTD measurements, which are high precision measurements of sound velocity (among other parameters).
- 540 expendable bathy termograph (XBT), which are temperature measurements with an expendable sensor. Sound speed is then calculated as a function of temperature and pressure. Variations of sound speed due to salinity variations are ignored. In this study, a constant value of 34.7 g/kg was used in the sound speed calculations.

Figure 1: Positions of XBT measurements (left panel) and CTD measurements (right panel).

The data were mainly from the Norwegian coast from Vestfjorden and northwards, and parts of the Barents Sea (Figure 1). A few measurements (108 XBT, 9 CTD) are from a small area outside the western coast of Norway. The data are from 2001 and 2002. Data for larger part of the coast can be made available within a few weeks notice. Due to time limitations, it was chosen to conduct the analysis with the data available at FFI at March 2003.

The sound speed gradient for a profile was calculated as follows: For each data point in a sound speed profile, the difference in sound speed between this data point and a point 5 meters below is calculated. If there is no data point exactly 5 m below, interpolation between the nearest available data points is used. The largest positive and negative sound speed gradient for each profile is retained and used for later analysis.
Figure 2: Example of sound speed profile (blue line) based on XBT measurement. The marked discontinuity at approximately 160 meters depth indicates that the conducting wire has broken when the sensor was at this depth. The data above the breaking point are valid data.

The data went through a certain subjective quality control. Especially the XBT data contains errors that highly affect the calculated maximum sound speed gradients. These errors arise mainly because the conducting wire between the temperature sensor and the vessel breaks during the measurements, rendering the data sampled beneath the breaking point useless. This is apparent as a discontinuity in the profile. An example of this is given in Figure 2.

The quality control consisted of visual inspection of the profiles with the largest sound speed gradient values. If a discontinuity is apparent, an appropriate maximum depth for this profile is defined. A few profiles were rejected in their entity. By this method, it is possible to eliminate the errors affecting our estimate of maximum sound speed gradient without manual inspection of 760 profiles.

Especially XBT measurements may contain errors in the uppermost part of the profile. This is due to thermal inertia of the sensor. The temperature sensor do no not have the time to adjust when passing from air to water when there is a large temperature difference between water and atmosphere, giving erroneous values in the uppermost data points. These errors are notoriously difficult to correct by visual inspection, as there often are large natural vertical temperature variations within the surface layer. No attempt has been done to correct for these errors.
2.1.2 Estimate of maximum values

![Figure 3: Sound speed profile (blue line). The maximum negative gradient over 5 m vertical distance in this profile occurs between 0 and 5 meters (marked with red).](image)

After the quality control, elementary statistic calculations were performed, providing maximum and minimum values for both positive and negative values of the sound speed gradient (Table 1, Appendix A).

By inspection, it was found that almost all of the largest sound speed gradients occurred within the uppermost part of the water column, and more often than not involved the uppermost data point of the profile. This data point is usually at 1 meter deep or less, which clearly is not representative for the submarines normal operational depth. A new minimum depth of 3 m was defined, and the calculations were performed again. This had the effect of reducing the positive maximum change in sound speed over 5 m vertical distance from +54 m/s to +9.4, and negative maximum values from –31.3 to –18.6 m/s (Table 2, Appendix A). The maximum values were also more evenly distributed over the depth of the water column. Figure 4 shows a profile with a maximum sound speed gradient of –16.0 m/s at approximately 20-25 m due to a large temperature gradient on the boundary between the surface layer and the water masses beneath.
2.2 Conclusion, typical maximum vertical variations in sound speed

As a reasonable maximum for vertical variations in sound speed over a vertical distance of 5 meter, the value $\pm 20$ m/s is chosen. This value us used in the sensitivity studies later in this report.

3 PASSIVE RANGING (PRS) METHOD

The method is based on measuring time of arrival at three hydrophones, hydrophone cluster, and computing the time differences shown in the figure 4 below.
Figure 5. Definition of parameters in the passive ranging method

- $T_{12}$ is the difference between time of arrival of the signal at hydrophone 1 and 2
- $T_{23}$ is the difference between time of arrival of the signal at hydrophone 2 and 3
- $a$ is the distance between hydrophones or hydrophone clusters

The bearing to the target is given by

$$
\theta = \arcsin \left( \frac{a}{2} \left( \frac{T_{12} + T_{23}}{c} \right)^2 \right)
$$

$$
\sin(\theta) = \frac{(T_{12} + T_{23})^2}{2a}
$$

and the range by

$$
R = \frac{a^2 \cos(\theta)^2}{c(T_{12} - T_{23})}
$$

In the calculations it is assumed that the sound speed is $c := 1480 \frac{m}{s}$

and $a := 150 \text{ m}$

We rewrite these equations and get
\[ T_{12} + T_{23} = \frac{2a}{c} \sin(\theta) \]
\[ T_{12} - T_{23} = \frac{a^2 \cos(\theta)^2}{Rc} \]

Adding and subtracting the equations we get
\[ T_{12}(R, \theta) := \frac{a}{c} \left( \frac{a}{2R} \cos(\theta)^2 + \sin(\theta) \right) \]
\[ T_{23}(R, \theta) := \frac{a}{c} \left( \frac{a}{2R} \cos(\theta)^2 - \sin(\theta) \right) \]

The sensitivity to variation in sound speed is given by
\[ \frac{d}{dc} T_{12} = -\frac{a}{c^2} \left( \frac{1}{2} \frac{a}{R} \cos(\theta)^2 + \sin(\theta) \right) \]
\[ \frac{d}{dc} T_{23} = \frac{a}{c^2} \left( \frac{1}{2} \frac{a}{R} \cos(\theta)^2 - \sin(\theta) \right) \]

or
\[ dT_{12}(R, \theta, dc) := -\frac{a}{c^2} \left( \frac{1}{2} \frac{a}{R} \cos(\theta)^2 + \sin(\theta) \right) dc \]
\[ dT_{23}(R, \theta, dc) := \frac{a}{c^2} \left( \frac{1}{2} \frac{a}{R} \cos(\theta)^2 - \sin(\theta) \right) dc \]

An error in sound speed of \( dc \) will therefore result in erroneous time difference measurements given by
\[ T_{e12}(R, \theta, dc) := T_{12}(R, \theta) + dT_{12}(R, \theta, dc) \]
\[ T_{e23}(R, \theta, dc) := T_{23}(R, \theta) + dT_{23}(R, \theta, dc) \]

and the error in computed range
\[ \Delta R(R, \theta, dc) := \frac{a^2 \cos(\theta)^2}{c(T_{12}(R, \theta) - T_{23}(R, \theta))} - \frac{a^2 \cos(\theta)^2}{c(T_{e12}(R, \theta, dc) - T_{e23}(R, \theta, dc))} \]  \( (1) \)

### 3.1 Error in the global sound speed

When the global sound speed is in error, the sound speed at the three hydrophones or hydrophone clusters all has exactly the same error. This results in an error in the range estimate given by equation 1. and plotted in figure 6. for \( dc := -20, -19, \ldots, 20 \)
Figure 6. Error in range estimate due to error in the global soundspeed from -20 m/s to 20 m/s for a target at 10000 m range and bearings 0, 20 and 40 degrees from broadside.

It is seen the the error in range as function of error in global sound speed is small and independent of bearing to the target.

3.2 Error in the sound speed along the array

If the sound speed along the array changes in an unknown manner the results may be different. Let us assume that the error in sound speed in front of the array has an estimation error of $-dc$, zero in the centre and $+dc$ at the end of the array. The error in range is now given by

$$
\Delta R(R, 0, dc) := T_{12}(R, 0) - dT_{12}(R, 0, dc)
$$

$$
\Delta R(R, 0, dc) := T_{23}(R, 0) + dT_{23}(R, 0, dc)
$$

$$
\Delta R(R, 0, dc) := \frac{a^2 \cos(\theta)^2}{c(T_{12}(R, 0) - T_{23}(R, 0))} - \frac{a^2 \cos(\theta)^2}{c(Te_{12}(R, 0, dc) - Te_{23}(R, 0, dc))}
$$

(2)

The result is shown in figure 7 for an error in sound speed $dc$ that varies from -10m/s to 10m/s $dc := -10, -9, .. 10$
It is seen that the error in range is very sensitive to differences in sound speed along the array.

3.3 Conclusion on PRS method

The positioning error is not very sensitive to errors in the global sound speed. However, the computed position is very sensitive to error in the sound speed along the array. Ideally the sound speed should be measured at each of the three sub-arrays. As a substitute the sound speed in front of the array could be monitored continuously and appropriately delayed versions of these measurements used as estimated sound speeds along the array. The accuracy of this method depends on how fast the sound speed varies in time and space. However, the extreme sensitivity to variations in the sound speed may make this method unreliable.
4 BEARING ERROR

Figure 8 shows a section of a towed array with hydrophones spaced $d$ meters apart. A signal comes in $\theta_0$ degrees from broadside with a wavefront shown in blue.

The steering is accomplished by delaying the signals from the $n$-th hydrophone by the time $n^* T_0$, where the correct $T_0$ computed for the sound speed $c_0$ by the equation

$$T_0 = \frac{d \sin(\theta_0)}{c_0}$$

If the sound speed is different the new steered angle will be

$$\theta = \text{asin} \left( \frac{T_0 c}{d} \right) = \text{asin} \left( \frac{c \sin(\theta_0)}{c_0} \right)$$

The error in angle is given by

$$\frac{d}{dc} \left( \text{asin} \left( \frac{c \sin(\theta_0)}{c_0} \right) \right) = \frac{\sin(\theta_0)}{c_0 \left( 1 - \frac{c^2 \sin(\theta_0)^2}{c_0^2} \right)^{1/2}}$$
\[
d\theta(\theta_0, dc) := \frac{\sin(\theta_0)}{c_0 \left( 1 - c^2 \frac{\sin(\theta_0)^2}{c_0^2} \right)^{1/2}} \cdot dc
\]  

(3)

If the global sound speed changes from -20 m/s to 20 m/s the resulting bearing error given by equation 3 is shown in figure 9

\[
dc := -20, -19, 20
\]

![Figure 9. Error in bearing in degrees as function of error in global sound speed from -20 m/s to +20 m/s](image)

Red line: broadside
Blue dotted line: 20 degrees off broadside
Green dashed line: 40 degrees off broadside
Red dot-dashed line: 60 degrees off broadside
Blue solid line: 80 degrees off broadside

It is seen that the error in bearing is small even with an error in sound speed of 20 m/s, and also that the error increases as the beam is steered away from broadside. This error does not increase linearly with steered angle away from broadside as shown in figure 10.
Figure 10. Error in bearing as function of steering angle from – 89 to 89 degrees from broadside.

Red curve: 5m/s error in sound speed
Blue dotted curve: 10m/s error in sound speed
Green dashed curve: 20m/s error in sound speed

It is clearly seen that the bearing error is less than 1.5 degrees when the beam is steered 60 degrees from broadside even with an error in sound speed of 20 m/s. At 80 degrees from broadside the bearing error increases to 5 degrees, which may seem large. But as figure 11 shows the beamwidth of a beam steered so far from broadside is much larger than 5 degrees. In reality a 20 m/s error in sound speed is therefore of little importance even at these extreme steering angles.
Figure 11. Beampatterns for a 31 element array with hydrophone spacing $0.9\lambda/2$.

- **Red**: beampattern at broad side
- **Black**: beampattern steered to 80 degrees off broadside
- **Blue dotted**: beampattern 80 degrees off broadside when the sound speed changes 20 m/s

## 5 TRIANGULATION USING THREE SUB-ARRAYS

### 5.1 Error in range as function of change in global sound speed

The question is now if the error in global sound speed will have the same detrimental effect on the range estimation as for the PRS method.

Let the range from the centre array (array 2) to the target be $R_0$, the range from array 1 $R_1$ and from array 3 $R_3$ and the distance between centre of two neighbour arrays “a” as in figure 5. Then
\[ R_0 \sin(\theta_2) = R_1 \sin(\theta_1) - a = R_3 \sin(\theta_3) + a \]

at long distance the three ranges are approximately equal. Therefore

\[ \theta_1 = \arcsin \left( \sin(\theta_2) + \frac{a}{R_0} \right) \]
\[ \theta_3 = \arcsin \left( \sin(\theta_2) - \frac{a}{R_0} \right) \]

When \( \theta_1 \) and \( \theta_3 \) are known the range can be found by the equation

\[ R_0 \sin(\theta_1) = R_0 \sin(\theta_3) + 2a \]

Substituting for \( \theta_1, \theta_3 \)

\[ R_0(R_0, \theta) := \frac{2a}{\sin \left( \arcsin \left( \sin(\theta) + \frac{a}{R_0} \right) \right) - \sin \left( \arcsin \left( \sin(\theta) - \frac{a}{R_0} \right) \right)} \]

Due to the error in the sound speed a bias is introduced in \( \theta_1 \) and \( \theta_3 \) resulting in an in range given by

\[ \Delta R(R_0, \theta, dc) := R_0(R_0, \theta) ... + \frac{-2a}{\sin \left( \arcsin \left( \sin(\theta) + \frac{a}{R_0} + d\theta(\theta, dc) \right) \right) - \sin \left( \arcsin \left( \sin(\theta) - \frac{a}{R_0} + d\theta(\theta, dc) \right) \right)} \]

The error in range estimate is shown in figure 12 for changes in sound speed from - 20m/s to 20 m/s
It is seen that if the sound speed at each sub-array is in error with the same amount the computed range is only changed by a small amount even at 10 km range.

### 5.2 Error in range as function of change in sound speed along the array

If the sound speed changes along the array the result may be different. This is here modelled by assuming that the sound speed at array no 1 is increased by $dc$ and the sound speed at array no 3 is decreased by $dc$, while the sound speed at the centre array is unchanged. The error is then given by

$$
\Delta R(\theta, dc) = R_0(\theta) \ldots + \frac{-2a}{\sin \left( \sin \left( \frac{a}{R_0} \right) + d\theta(\theta, dc) \right) - \sin \left( \sin \left( \frac{a}{R_0} \right) - d\theta(\theta, dc) \right)}
$$

The result is shown in figure 13 when the sound speed is changed from -10 to 10 m/s, equivalent to maximally 20 m/s change in sound speed along the array.

dc := -10, -9, 10

---

*Figure 13 Error in range estimate as function of error in sound speed along the array for a target at 10 km range.*

- **Red solid curve**: 40 degrees off broadside
- **Blue dotted curve**: 20 degrees off broadside
- **Green dashed curve**: 10 degrees off broadside
- **Horizontal curve**: broadside
By comparing figure 13 with the results for the PRS method, figure 7, it is seen that the method of Triangulation is much less sensitive to errors in the sound speed. It may therefore be sufficient to measure the sound speed in front of the array continuously and use appropriately delayed versions of these measurements to estimate sound speeds along the array.

### 6 SOUND SPEED AND TEMPERATURE

Wilson simplified model, accurate to 0.1m/s for T<20 deg C, and depth Z<8000 m. Is given by the following formula from URICK [1]

\[
c(T, S, Z) := \left[ 1492.9 + 3(T - 10) - 6 \cdot 10^{-3} (T - 10)^2 - 4 \cdot 10^{-2} (T - 18)^2 \cdots \right] \\
+ 1.2 (S - 35) - 10^{-2} (T - 18) (S - 35) + \frac{Z}{61}
\]

- S salinity in parts per thousand (ppT)
- T temperature in centigrade
- Z depth in metres

The sensitivity to temperature is given by

\[
\frac{d}{dT}c(T, S, Z) = \frac{491}{100} - \frac{23}{250} T - \frac{1}{100} S
\]

\[
dc(T, dT, S, Z) := \left( \frac{491}{100} - \frac{23}{250} T - \frac{1}{100} S \right) dT
\]

If the salinity is kept at 35 and depth at 100 m the variation in sound speed as function of temperature is shown in figure 14

- Z := 100
- T := 0.. 10

![Figure 14 Sound speed as function of frequency at 100 m depth and salinity 35 ppT](image-url)
The variation in sound speed width temperature around 8 degrees Centigrade is shown in figure 15:
\[ dT := -1, -0.9, 1 \]

![Graph showing change in sound speed as function of change in temperature around 8 degrees Centigrade at 100 m depth.](image)

**Fig 15** Change in sound speed as function of change in temperature around 8 degrees Centigrade at 100 m depth.

Fig 15 shows that a temperature shift of 0.25 degrees changes the sound speed 1 m/s. The temperature sensor should therefore have a resolution better than 0.25 degrees Centigrade. The absolute accuracy is not very critical, as neither of the methods are very sensitive to changes in the global sound speed.

### 7 CONCLUSION

The following conclusions can now be made:
- The bearing of a target is not very sensitive to changes in the sound speed. If the array is only used to measure bearing to a target, it will be sufficient to measure the sound speed in the tower of the submarine.
- The method of positioning the target by measuring the time of arrival at three hydrophones or hydrophone clusters (PRS method) is not very sensitive to a global error in the sound speed, but extremely sensitive to changes in the sound speed along the array, if this is not measured correctly and compensated for. Measuring the sound speed in the tower of a submarine may therefore not be adequate. Even measuring the sound speed at the same depth as the array may not be sufficient. Ideally the sound speed should be measured at each of the three hydrophone or hydrophone clusters along the array.
- The method of positioning the target by measuring the bearing at three sub-arrays along the array is not sensitive to error in the global sound speed, but sensitive to changes in the
sound speed along the array. This sensitivity is, however, much smaller than for the PRS method. It may therefore be sufficient to continuously measure the sound speed at the same depth as the array together with the speed of the submarine. Properly delayed sound speed measurements may be sufficiently accurate to be used for improving the cross bearing accuracy.

- The sound speed should be measured with a resolution better than 1 m/s. The absolute accuracy is not very critical as the estimated range is not very sensitive to changes in the global sound speed.

- If a temperature sensor is used the temperature resolution should be better than 0.25 degrees. The absolute accuracy is not very critical as the estimated range is not very sensitive to changes in the global temperature, which affects the global sound speed.

- If the PRS method is used the sound speed should be measured with resolution better than 0.25 m/s equivalent to a temperature resolution of 0.6 degrees Centigrade.
### A  STATISTICS FOR VERTICAL SOUND SPEED GRADIENTS OVER 5 METERS

#### Table 1. Statistics for maximum sound speed gradient over a vertical distance of 5 meters.

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<th>CTD data:</th>
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<td>dc / dz &gt; 0:</td>
<td>dc / dz &lt; 0:</td>
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<tr>
<td>max</td>
<td>8.14001</td>
<td>-31.3403</td>
</tr>
<tr>
<td>mean</td>
<td>1.81368</td>
<td>-10.4427</td>
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<tr>
<td>median</td>
<td>1.48181</td>
<td>-9.67029</td>
</tr>
<tr>
<td>min</td>
<td>0.117188</td>
<td>-0.511230</td>
</tr>
<tr>
<td>std.dev</td>
<td>1.31881</td>
<td>4.66550</td>
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|                | dc / dz > 0:                   | dc / dz < 0:                   |
| max            | 54.6001                        | -16.0000                       |
| mean           | 4.45683                        | -2.40533                       |
| median         | 1.30005                        | -1.09998                       |
| min            | 0.100098                       | -0.0999756                     |
| std.dev        | 9.87473                        | 2.87484                        |

#### Table 2. Statistics for maximum sound speed gradient over a vertical distance of 5 meters for depths greater than 3 meter.

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<td>max</td>
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<td>min</td>
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<td>std.dev</td>
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<td>4.05655</td>
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|                | dc / dz > 0:                   | dc / dz < 0:                   |
| max            | 9.40002                        | -16.0000                       |
| mean           | 1.47228                        | -2.43675                       |
| median         | 1.20007                        | -1.09998                       |
| min            | 0.0999756                      | -0.0999756                     |
| std.dev        | 1.25567                        | 2.88768                        |

Note: The leap in maximum values from Table 1.
References

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### FORDELING GODKJENT AV FORSKNINGSSJEF

| John-Mikal Størdal |

### FORDELING GODKJENT AV AVDELINGSSJEF:

| Johnny Bardal |

### EKSTERN FORDELING

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### INTERN FORDELING

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**Elektronisk fordeling:**

- FFI-veven
- Elling Tveit (ETv)
- Jarl Johnsen (JKJ)
- Trond Jenserud (TJe)
- Tommy Torgersen (TTo)
- Knut Søstrand (KAS)
- Elin M Dombesteen (EMD)
- Ellen J Eidem (EJE)
- Jon Wegge (JWe)
- Erik Sevaldsen (ESe)
- John-Mikal Størdal (JMS)
- Stig Lødøen (SEL)
- Arvid Melkevik (AMk)
- Harald Nyberg (HAn)
- Carina N. Vooren (CNV)
- Arne Cato Jenssen (ACJ)
- Åslaug Grovlen (AGr)

---

.R. FFI-K1  